Optimal Vibration Control of a Model Frame Structure Using Piezoceramic Sensors and Actuators

V. SETHI
G. SONG
Department of Mechanical Engineering, University of Houston, 4800 Calhoun Road, Houston, TX 77204-4006, USA (gsong@uh.edu)

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Abstract: This research concerns the active vibration control of a three-story model structural frame by using piezoceramics smart materials, in particular lead zirconate titanate (PZT), in the form of patches. These PZT patches are surface-bonded on the structure and function as actuators and sensors. To assist the control design, system identification is first performed. To excite the frame for identification purposes, a sweep sinusoidal input is used to drive the PZT actuator patches, and then a sixth-order state space model with a fit of 90% is identified to represent the dynamics of the first three modes of the model structural frame. A full-state linear quadratic regulator (LQR) controller is designed using the identified model. To achieve the full state feedback, an observer is designed based on the identified model. The LQR controller along with the observer is implemented to actively control the vibration of the model frame structures and is experimentally verified effective in the simultaneous suppression of vibrations at the first three modes.

Key Words: Active vibration control, piezoceramics, building control, civil structural control, linear quadratic regulator control, observer

1. INTRODUCTION

Structural vibration control has always received considerable research attention from the civil engineering community. In this regard, an excellent review of structural control applied to civil engineering by Housner et al. (1997) provided special insight into various requirements of structural control for civil engineering and laid down the future research needs in this potentially growing field. Ever since research in this field was initiated, there has been continuous research in structural control either by active, passive, semi-active or hybrid vibration control methods. Passive vibration control using tuned passive mass dampers has been used in buildings for improved structural performance. Soong (1988) reviewed the importance and necessity of active vibration control in civil engineering. Since this concept was non-traditional for civil engineering, real obstacles with respect to its acceptance existed at that time (Soong, 1988).

Active mass tuned dampers have also been installed in buildings (Wu et al., 1995) and television towers (Cao et al., 1998) and have been successful because they can produce tons of forces necessary for the structural response control of the buildings. However, active/passive mass dampers have limitations too. They have high inherent costs and, besides

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this, the associated costs of the driving power source are also high (Valliappan and Qi, 2001). To overcome the above limitations, the answer may lie in intelligent or smart structures.

Intelligent or smart or adaptive structures are a subset of active structures that have highly distributed actuator and sensor systems with structural functionality, and in addition distributed control function and computing architecture (Crawley, 1994). Smart materials can be used as actuators and sensors, and include piezoelectric materials, shape memory alloys, electrostrictive materials, magnetostrictive materials, electrorheological, magnetorheological fluids and fiber optics.

Structural vibration control using smart materials is being increasingly used for flexible structures in the aerospace industry. Over the last decade, the use of piezoceramics as actuators and sensors has considerably increased, and they provide effective means of high-quality actuation and sensing mechanism. Piezoceramics have also been known as low-cost, lightweight, and easy-to-implement materials for the active control of structural vibration.

Initial research into piezoelectric materials for use in intelligent structures was carried out by Crawley and de Luis (1987). The active vibration control of a cantilever beam using piezoelectric polymer poly vinylidene fluoride (PVDF) was investigated by Bailey and Hubbard (1985). Lazarus and Crawley (1992) demonstrated the induced strain actuators for use as high control authority linear quadratic Gaussian (LQG) multi-input multi-output (MIMO) control on flexible plate-like structures. Hagood and Anderson (1992) addressed the issue of simultaneous sensing and actuation using piezoelectric materials. Scott et al. (2001) used pole placement control for control of lightly damped structures. Manning et al. (2000) and Bu et al. (2003) have used the system identification ARMAX model and pole placement for vibration control of a beam. Han et al. (1997) used the modeling techniques and LQG control of composite plate.

Butler and Rao (1996) presented a structural identification technique based on the measurement of eigenvalues and eigenvectors of the structure for implementation of MIMO full state feedback controllers. To model each mode, two sensors are needed, thus requiring a large number of sensors for the implementation of such a model but significantly reducing the computing effort. Sethi et al. (2002) and Sethi (2002) implemented system identification and positive position feedback and strain rate feedback control on a large composite I-beam for vibration suppression. Chen and Shen (1997) proposed the independent modal space control method to perform optimal control and the controllability and observability of specified modes for surface-bonded piezoelectric structures. Prakah-Asante and Craig (1994) presented the LQG method for multichannel control of vibration transmission of disturbances in finite beam structures bonded with piezoceramics. Kamada et al. (1997) used lead zirconate titanate (PZT; a type of piezoceramic) stack actuators for active control of a frame structure with the model matching and *H* infinity methods.

In this paper we present exploratory research of the active vibration control of frame structures using piezoceramic patches with a three-story model building. PZT patches are surface-bonded on the structure at appropriate locations and function as actuators and sensors. The control design of flexible structures relies on accurate modeling of the system dynamics, in the absence of adaptive or robust control (Saunders et al., 1994). In this paper, to facilitate the control design of a linear quadratic regulator (LQR) controller for multimodal vibration suppression of the frame structure, system identification and state estimation are first carried out. The LQR controller along with the state estimator is implemented to actively control the vibration of the model frame structures and is experimentally verified effective.

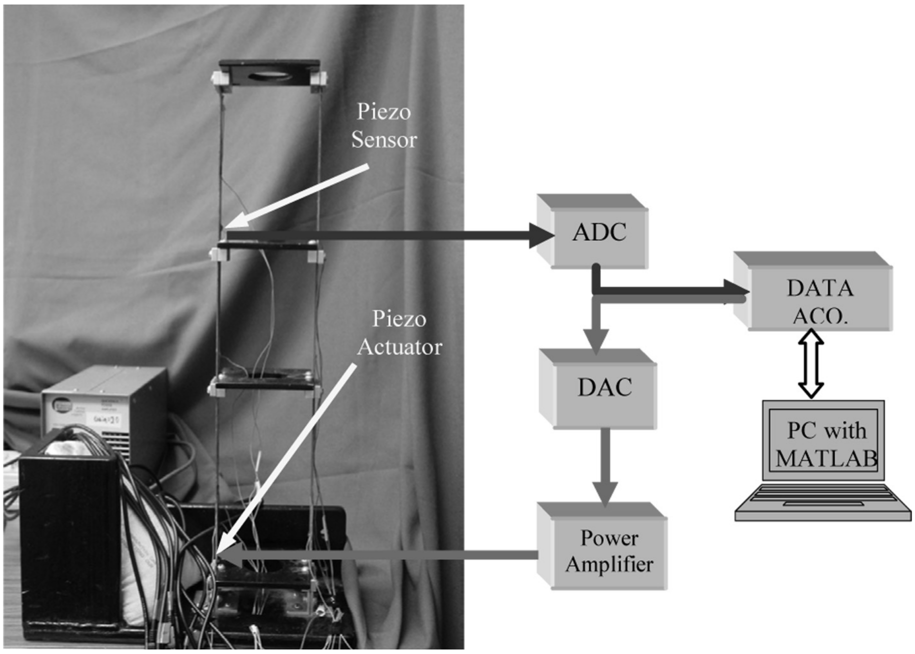

Figure 1. Experimental setup.

Table 1. Building model dimensions.

Symbol	Quantity	Units	Value
L	Length	mm	535
w_b	Width	mm	108
t	Thickness	mm	30
t_b	Thickness of the beam	mm	2

2. EXPERIMENTAL SETUP

The control objective is to optimally suppress the vibrations of a three-floor model building by using smart sensors and actuators. The setup is shown in Figure 1 and the dimensions are listed in Table 1. Two larger PZT-5H patches are surface-bonded at either side of the support beam near its base end. These PZT patches are used as actuators to excite the model building and to enable active control of the induced vibration. There is also one PZT patch, surface-bonded at the bottom of the third floor, which acts as a sensor. The sensor provides the feedback signal in the active control algorithm. The properties of PZT are shown in Table 2. In this experiment, there is one input for the two actuators and one output was recorded from the sensor.

For implementing the controller in real time, a dSPACE digital data acquisition and real-time control system are used. The dSPACE uses a DS1103 digital signal process board for

Table 2. PZT-5H patch properties.

Symbol	Quantity	Units	PZT actuator	PZT sensor
$L \times w \times t$	Dimensions	mm	$70 \times 30 \times 0.25$	$15 \times 12 \times 0.25$
d_{33}	Strain coefficient	C N^{-1}	5.93×10^{-10}	5.93×10^{-10}
d_{31}	Strain coefficient	C N^{-1}	-2.74×10^{-10}	-2.74×10^{-10}
ρ_p	PZT density	Kg m^{-3}	7500	7500
E_p	Young's modulus	N m^{-2}	6.3×10^{10}	6.3×10^{10}

real-time control implementations. The dSPACE system also has integrated analog-to-digital and digital-to-analog converters.

3. SYSTEM IDENTIFICATION

3.1. Theoretical Concepts

The approach to determining the transfer function using the mathematical modeling and finite element analysis is complex. The finite element models sometimes are not feasible for control purposes because the order is high for achieving the desired accuracy. However, the system identification technique based upon experimentation offers a rather simplistic approach for obtaining the transfer function of the system. Based upon the input, the output signals from the system are analyzed in order to obtain a model.

In this paper, the system identification algorithm used to identify the system is based upon the subspace method. A linear system can be represented in the state space innovations form as

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + Ke(t) \\ y(t) &= Cx(t) + Du(t) + e(t) \end{aligned} \tag{1}$$

where $e(t)$ is the innovation (i.e. the output that cannot be predicted from the past data), $x(t)$ is the state vector, $y(t)$ is output, $u(t)$ is the input, and K is the Kalman gain.

The subspace method can be used to estimate the A, B, C, D, and K matrices. Assuming that $x(t)$, $y(t)$, and $u(t)$ are known, equation (1) becomes a linear regression. This will enable us to estimate the matrices C and D by the least-squares method and will lead us to determine $e(t)$.

Again, $e(t)$ can be treated as a known signal and this will lead us to determine A, B, and K using the least-squares method. The Kalman gain K is computed using the Riccati equation. In the above method, initially it is assumed that states $x(t)$ are known; however, they need to be determined. The states $x(t)$ can be formed as linear combinations of the k step ahead predicted outputs. The predictor, in this method, can be determined using the k step ahead predictors by projections from the observed data sequences. The above model derived from the subspace method is then used as the base model for further refining model by the prediction error method (PEM).

In the time domain, the above system can be represented by using the shift operator q as

$$y(t) = G(q)u(t) + H(q)e(t) \tag{2}$$

$$G(q) = C(qI - A)^{-1}B + D \tag{3}$$

$$H(q) = C(qI - A)^{-1}K + I \tag{4}$$

where $G(q)$ is the transfer function of the system, $e(t)$ is the innovation, and I is the identity matrix.

From the observed data of input u and output y, the prediction errors can be computed as

$$e(t) = H^{-1}(q)[y(t) - G(q)u(t)]. \tag{5}$$

The above error can now be parametrized by the state space matrices derived above by the subspace method. The common parametric identification method is to determine estimates of G and H by minimizing

$$V_N(G, H) = \sum_{t=1}^{N} e^2(t). \tag{6}$$

This forms the basis for the PEM. The model is first initialized and further adjusted by optimizing the prediction error fit. Substantial details for system identification can be found in Ljung (1999) and the MATLAB System Identification Manual (http://www.Mathworks.com). MATLAB has the system identification toolbox to perform the above algorithm. The PEM first initializes the model by using the subspace algorithm and then minimizes the prediction error.

In this paper, we consider a state space realization that does not model the noise properties, i.e. an output error model ($K = 0$). Thus, the implications will be on the predictors, which will be based on the past inputs only.

3.2. Experimental Identification Results

Using the discussed experimental setup, the piezoactuators are excited by a sweep sinusoidal signal of frequency varying from 1 to 75 Hz in a sweep time of 200 s. The actuators signal is amplified using a power amplifier and the input voltage is set at 120 V. The samples are collected at a frequency of 1000 Hz for 400 s so as to collect substantial data for model parametrization. The data collected are detrended and used for the evaluation and validation of the model. Figure 2 shows the output signal from the structure or the total data set collected for evaluation and validation by excitation of the structure using a sweep sine signal. Using the MATLAB System Identification toolbox, a state space model of sixth order is obtained. Similar to the evaluation data, another experiment was performed to capture the validation data. Figure 3 shows the validation results of the identified model with a fit of 90%. The frequency response in Figure 4 of the identified model clearly shows the resonance frequencies at 7.70 Hz (48.4 rad s^{-1}), 24.03 Hz (151.04 rad s^{-1}) and 45.67 Hz (286.95 rad s^{-1}). Table 3

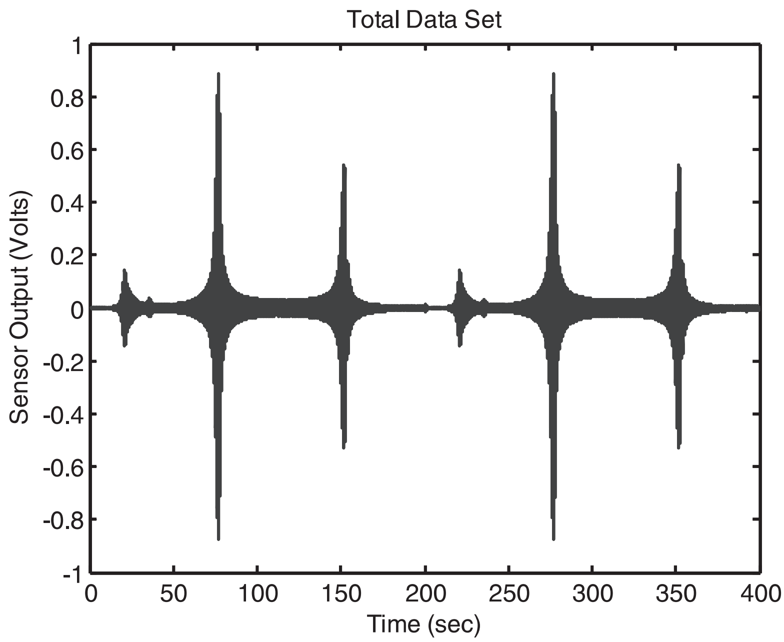

Figure 2. Total data set for evaluation and validation.

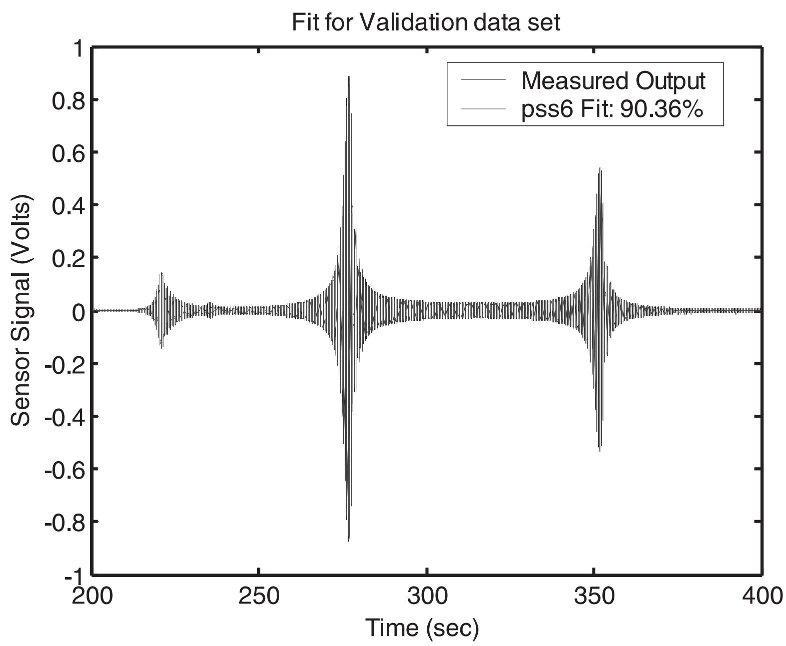

Figure 3. Identified model fit for validation data set.

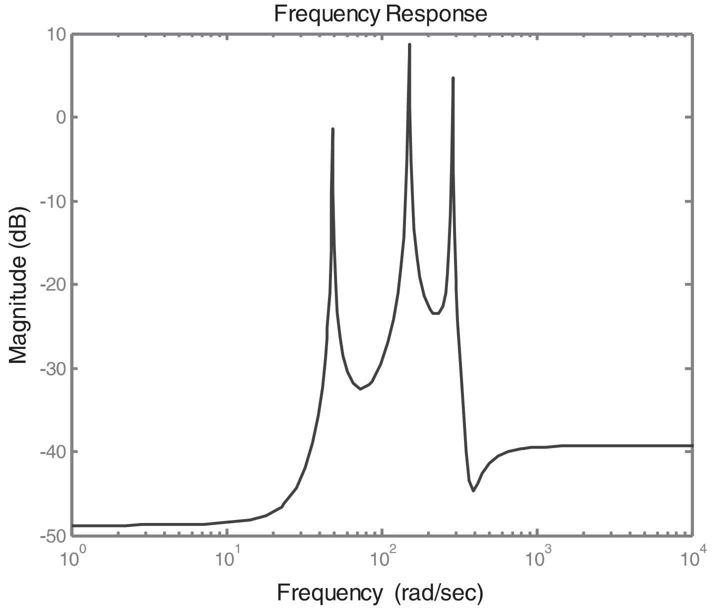

Figure 4. Frequency response of the identified model.

Table 3. Frequency comparison.

	Measured frequency	Identified model frequency
First mode	7.66 Hz	7.70 Hz
Second mode	23.89 Hz	24.03 Hz
Third mode	45.81 Hz	45.67 Hz

compares the measured (experimental) frequencies of the model building with that of the identified model.

4. OPTIMAL CONTROLLER AND STATE ESTIMATOR

In this research, a linear quadratic optimal controller is designed to control the first three modes of the flexible structure. A continuous time state space representation of the system is given by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du. \end{aligned} \tag{7}$$

Once the matrices A, B, C, and D are known, the task left is to determine the optimal controller u so as to minimize the objective quadratic function given by

$$J = \int_0^\infty \left[x^T Q x + u^T R u\right] dt \tag{8}$$

$$u = -Kx. \tag{9}$$

Here Q is a state weight matrix, R is a control weight matrix, and K is the optimal controller gain. To achieve quick vibration suppression, a larger value of the state weight matrix Q can be chosen or, in other words, the penalty on state is increased. Also, to reduce the energy consumption, the penalty on u can be increased by choosing a larger value of R matrix. The optimal controller gain K can be determined using the steady-state Riccati equation (10)

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{10}$$

$$K = R^{-1}B^T P \tag{11}$$

where P is the solution of the Riccati equation.

It should be noted that in the above optimal controller u the feedback is of the full state x of the system. However, in a practical situation we may not always be able to obtain the entire states of the system, and often the output is a possible combination of all the states. In such situations we need a state estimator or observer to obtain full states of the system.

The design of the state estimator can be done using the Kalman estimator or by standard pole assignment techniques. Here, an estimator based on standard pole placement techniques is utilized. The dynamics of estimator are given by

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + L(y - \hat{y}) \\ \hat{y} &= C\hat{x} + Du \end{aligned} \tag{12}$$

where L is the estimator gain that determines the convergence of $x \to \hat{x}$. The estimator gain can be found using the conventional Ackermann formula by forming the dual of the system given by equation (7):

$$\dot{x} - \dot{\hat{x}} = Ax + Bu - A\hat{x} - Bu - L(y - \hat{y}). \tag{13}$$

Defining $\varepsilon = x - \hat{x}$

$$\begin{aligned} \dot{\varepsilon} &= \dot{x} - \dot{\hat{x}} = A(x - \hat{x}) - L(Cx + Du - C\hat{x} - Du) \\ \dot{\varepsilon} &= (A - LC)\varepsilon. \end{aligned} \tag{14}$$

Thus, if we put the observer in a closed-loop system, the optimal controller will be modified as

$$u = -K\hat{x}. \tag{15}$$

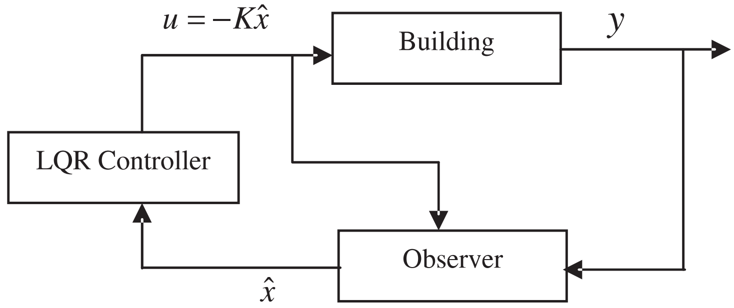

Figure 5. Block diagram with observer.

The closed-loop system can be written as

$$\begin{aligned} \dot{x} &= Ax - BK\hat{x} = (A - BK)x + BK(x - \hat{x}) \\ \dot{\varepsilon} &= (A - LC)\varepsilon. \end{aligned} \tag{16}$$

In matrix form, the above closed-loop system is summarized as

$$\begin{bmatrix} \dot{x} \\ \dot{\varepsilon} \end{bmatrix} = \begin{bmatrix} (A - BK) & BK \\ 0 & (A - LC) \end{bmatrix} \begin{bmatrix} x \\ \varepsilon \end{bmatrix}. \tag{17}$$

The eigenvalues of the above matrix are the eigenvalues of $(A - BK)$ and the eigenvalues of $(A - LC)$. From the above, it is clear that we can choose the optimal gain K and observer gain L independently.

For effective vibration suppression, it is necessary to choose the weighting matrices Q and R appropriately as they represent the importance between the control effect and control effort. The weighting matrices chosen are

$$Q = \text{diag}(10,\ 10,\ 0.2, 0.\ 1, 1,\ 0.5)\ \text{ and } R = 1.$$

The matrix Q is chosen by trial and error and by observing its effect on the time response and power spectral density of the sensor signal. The states x_1 and x_2 have higher weight because they correspond to the third mode, and the states x_3 and x_5 correspond to the second and first modes, respectively, in the identified model. The above weighting matrices when substituted in equation (10) will help us to obtain optimal gain K using equation (11). Thus, now we need to obtain the states of the system for feedback law. These states can be obtained by the observer for which we use the pole assignment approach. Pole assignment provides the designer with a flexibility to choose the poles of the system so as to obtain the desired performance characteristics. Graphically, the closed-loop system with observer is shown in Figure 5.

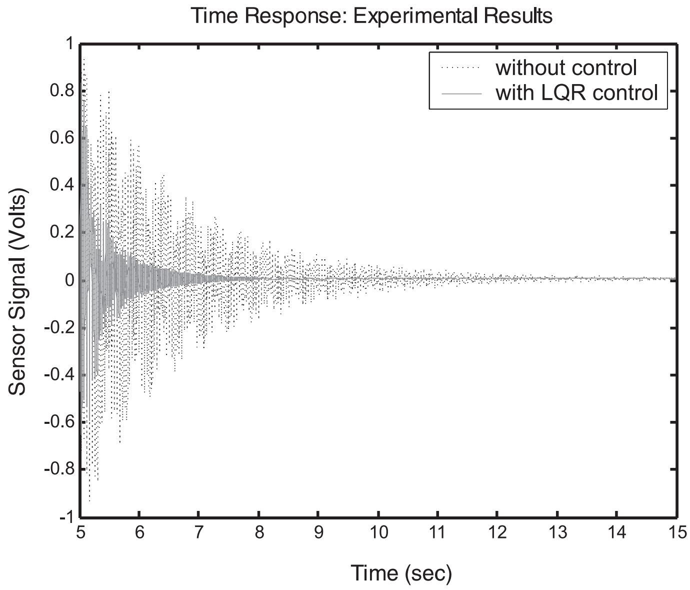

Figure 6. Comparative time response.

5. EXPERIMENTAL RESULTS OF MULTIMODE OPTIMAL VIBRATION CONTROL

The model found above in system identification coupled with the optimal controller is implemented in a real-time configuration. The building structure is excited using three sinusoidal signals of modal frequencies for an initial 5 s at 40 V each, coupled with a noise input. The controller was implemented in a real-time data acquisition system with a sampling frequency of 1 KHz. The open-loop test is conducted to view the results in the absence of a controller, and the time response is shown in Figure 6. Next, the controller is implemented. The real-time LQR controlled time response is shown in Figure 6. The uncontrolled vibrations that took about 10 s to die are now suppressed within 3 s. The power spectral density comparison plots for 6–12 s of data are shown in Figure 7. The power spectral density from 6 to 12 s is considered because the sudden kicking of the control action after the excitation at 5 s might lead to the excitation of higher modes, which die down because of their own damping over the next second. Evidently, substantial drops are observed simultaneously in the first three modes of the structure.

To implement the controller in real time, low pass and band pass filters are used. This is done to prevent the spillover effects. The piezoelectric actuators have a tendency to excite the higher-order modes more strongly than other actuation mechanisms. Therefore, it becomes imperative to employ filters. In the absence of filters, the spillover effect was observed.

The observer results in the free vibration case for the first three modes are also shown in Figures 8, 9, and 10, respectively. From these figures, it can be seen that the observer

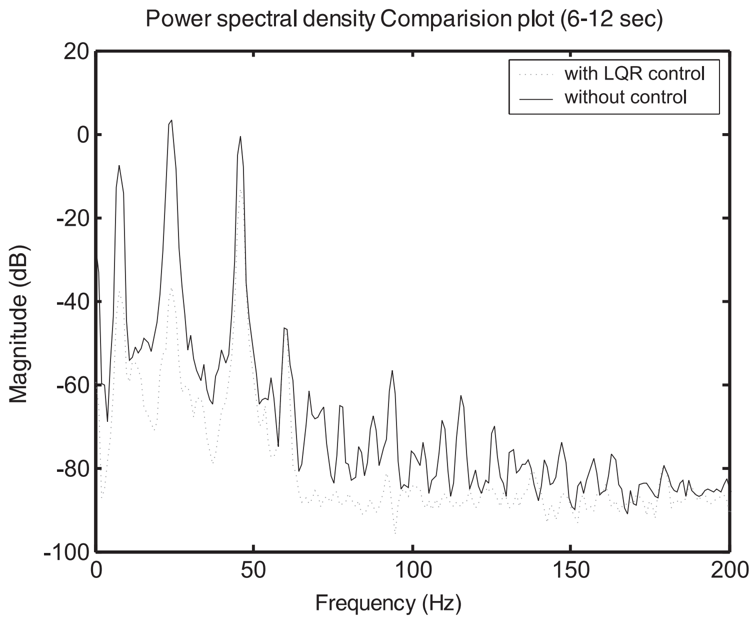

Figure 7. Comparative power spectral density plots.

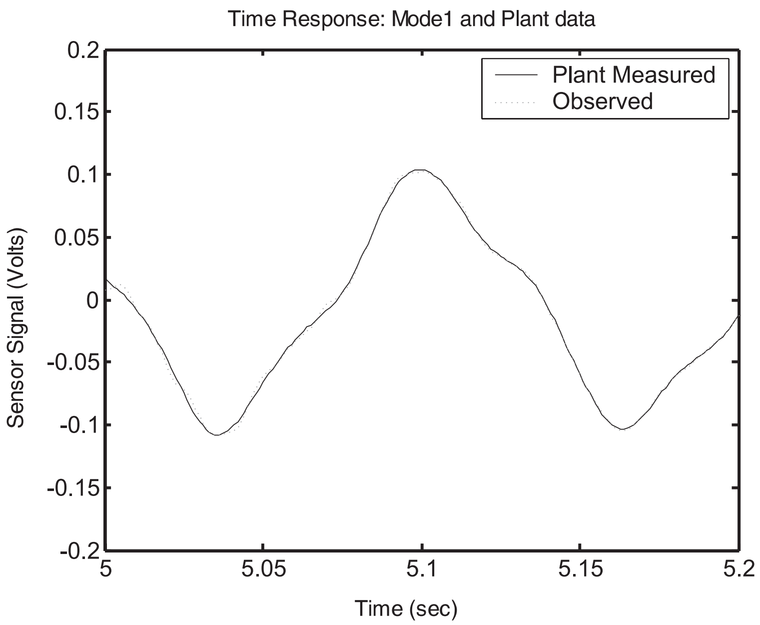

Figure 8. Time response comparison from observer and plant for the first mode in free vibration.

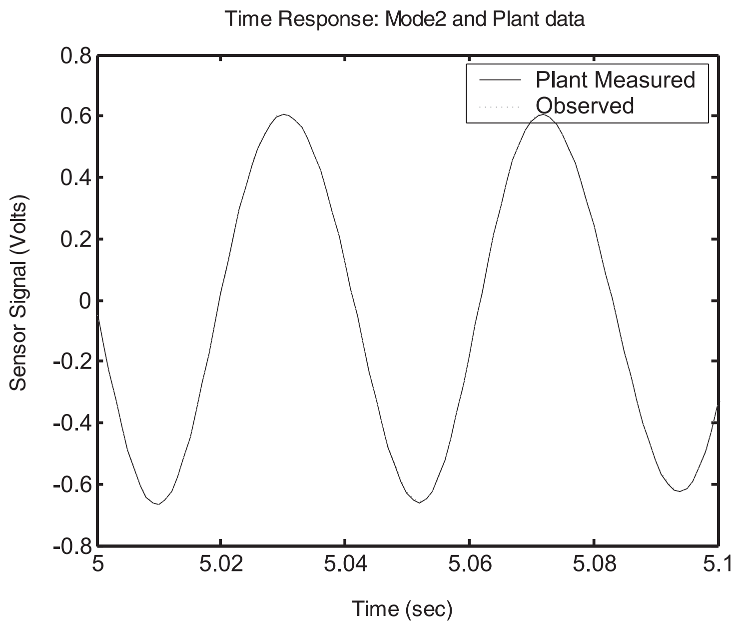

Figure 9. Time response comparison from observer and plant for the second mode in free vibration.

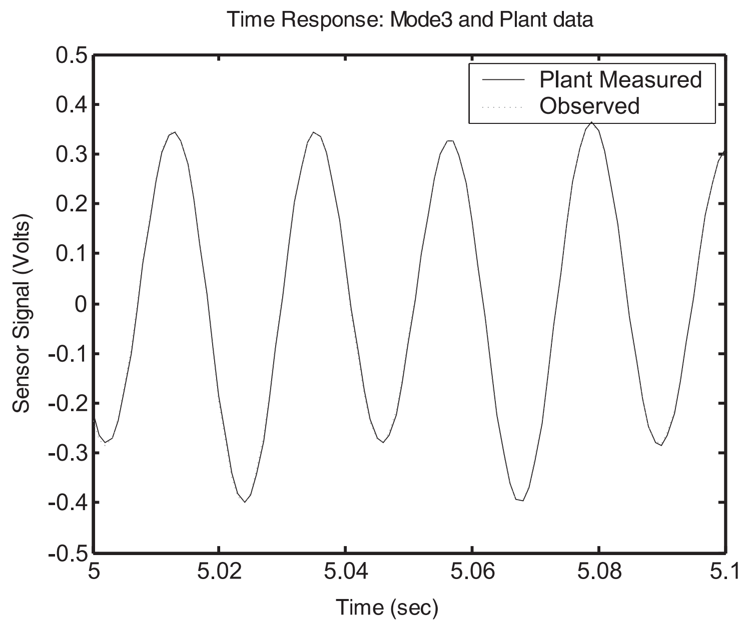

Figure 10. Time response comparison from observer and plant for the third mode in free vibration.

outputs closely match with the plant output for the respective modes. The plant outputs for the respective modes were acquired using band pass filters.

6. CONCLUSIONS

This experiment has successfully demonstrated simultaneous multimode vibration suppression of a building structure using piezoceramics. The system identification coupled with observer design and LQR control represents the completeness of the control system design. The model obtained using system identification with a 90% fit accurately represents the dynamics of the system as verified experimentally. The observer design gives the full states of the system accurately for use in the feedback control design. The observer output for the respective modes depicted in Figures 8, 9, and 10 confirms that the observer output converges to the true plant output in a few tenths of a second. Finally, the LQR control, with weights placed on individual states, proves to be very effective in multimodal vibration suppression. The power spectrum density plot in Figure 7 clearly demonstrates its effectiveness for multimode control.

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