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Active Vibration Suppression of a Smart Flexible Beam Using a Sliding Mode Based Controller

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Abstract: This article investigates robust active vibration suppression of a flexible beam with a low dominant frequency using piezoceramic sensor and actuators. The piezoceramic sensor and actuators are in patch form and are surface-bonded to the cantilevered end of the flexible beam. The robust sliding mode control, which has the advantages of being robust to plant parameter variation, insensitive to the unmodeled dynamics, and easy to implement, is adopted in this article for active vibration control of the flexible beam. Unlike other commonly used vibration suppression methods, such as positive position feedback control, strain rate feedback control, and lead compensation, the sliding mode controller requires almost no knowledge of the flexible beam. To avoid the chattering problem commonly associated with sliding mode control, a smooth switching function employing a hyperbolic tangent function is used. A low pass filter is applied to the output of the sliding mode controller to avoid excitation of the higher modes of the flexible beam. To demonstrate the advantage of sliding mode based active vibration reduction, its experimental results are compared with those of Proportional plus Derivative (PD) control and lead compensation. The comparison shows that the sliding mode controller reduces vibration of the flexible beam much more rapidly. To verify the robustness of the proposed sliding mode controller, vibration suppression of the beam is conducted for a case where the modal frequency of the beam is changed by adding masses. In addition, vibration suppression of the beam when subjected to a multi-modal excitation is also conducted. Experimental results demonstrate the robustness of the proposed controller with respect to varying model parameters and even the dynamics of higher modes.

Key words: Robust control, sliding mode control, vibration control, piezoceramic sensor and actuator

1. INTRODUCTION

Recent years have seen the emergence of the new field of active structural control using smart materials. Among the smart materials used in active structure control, piezoceramics have proven to be among the most promising active materials. The advantages of piezoceramics include low power consumption, absence of moving parts, rapid response, compactness and easy implementation. In addition, piezoceramics can be used effectively as both sensors and actuators. Due to these advantages, piezoceramic materials have been used in active vibration control of a variety of structures (see, e.g., Goh and Caughey, 1985; Fanson and Caughey, 1990; Agrawal and Bang, 1994; Song et al., 2000; Schirmacher, 1999; Proulx and Cheng, 2000).

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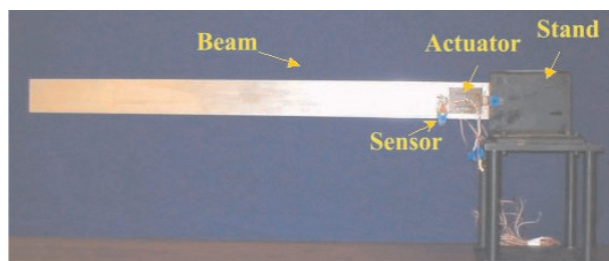


Figure 1. Experimental set up.

There are often uncertainties such as inexactly known structural mass or stiffness. This requires the controller for active vibration suppression to be robust with respect to such uncertainties. The sliding mode method is an effective method of dealing with uncertainties in both linear and nonlinear dynamic systems. This method has the advantages of being robust to un-modeled nonlinearities, improving transient performance and improving control accuracy. Reported cases of vibration control using the sliding mode method include wind and seismic response control of a building (Yang et al., 1997), the control a large flexible structure using jet thrusters (Allen et al., 2000), the vibration control of tall buildings using an actively tuned mass damper (ATMD) (Adhikari and Yamaguchi, 1997), and active vehicle suspension, with the sliding surface obtained using linear quadratic control theory (Yoshimura et al., 2001) or using a time-varying sliding surface (Cai and Song, 1994). However, sliding mode based active vibration control using piezoceramic actuators is rarely reported.

In this article, a sliding mode controller is designed for the active vibration control of a flexible aluminum beam using a piezoceramic patch sensor and actuators. Other types of active control methods, such as traditional PD control and lead compensation, are applied to the beam for comparative study. Experimental results clearly demonstrate that the sliding mode control suppresses the beam's vibrations more effectively than both the PD control and the lead compensator. Further experimental results show the robustness of the sliding mode controller to the varying of the model parameters of the flexible beam. A multi-mode excitation experiment is also conducted, to verify the robustness of the sliding mode controller to the dynamics of higher modes.

2. EXPERIMENTAL SETUP AND OPEN LOOP TESTING

2.1. Beam and Piezoceramic Sensor and Actuator Properties

An aluminum beam in a cantilevered configuration is used as the experimental object, to test the effectiveness of various active vibration controls. The experimental setup is shown in Figure 1. The properties of the beam are given in Table 1. PZT (lead zirconate titanate) piezoelectric ceramic patches are used as the smart sensor and actuator, and are surface-bonded to the beam. Two PZT patches are bonded to the beam near its cantilevered end, one on each side of the lateral surface of the beam. These two patches are used as actuators to excite the beam and enable active vibration control. An additional PZT patch is bonded on

Table 1. Beam properties.

Notation	Description	Units	Value
L	Beam length	mm	736.5
w_b	Beam width	mm	53.1
t_b	Beam thickness	mm	1
ρ_b	Beam density	kg/m ³	2690
E	Modulus of elasticity	N/m ²	7.03×10^{10}

Table 2. Properties of PZT patches used on the beam.

Notation	Description	Units	PZT Actuator	PZT Sensor
$L \times w \times t$	Dimensions	mm	$46 \times 33.27 \times 0.25$	$14 \times 7 \times 0.25$
d_{33}	Strain coefficient	C/N	7.41×10^{-10}	7.41×10^{-10}
d_{31}	Strain coefficient	C/N	-2.74×10^{-10}	-2.74×10^{-10}
ρ_P	PZT density	Kg/m ³	7500	7500
E_P	Young's Modulus	N/m ²	6.3×10^{10}	6.3×10^{10}

one lateral surface of the beam, also near its cantilevered end. This PZT patch acts as a sensor to measure the local strain, and the strain is used for feedback control in the active vibration suppression algorithms. The properties of the PZT patches are given in Table 2.

2.2. Open Loop Testing of the Aluminum Beam

First, open loop testing is performed to find the dominant mode of the beam for active vibration control. The beam is subjected to an impact, producing a multi-mode excitation. The time response of the PZT sensor is shown in Figure 2, and it can be seen that higher-modal vibrations damp out after 7.5 s, after which only first mode vibration remains. A power spectrum density (PSD) plot of this vibration data reveals the modal frequencies of the system: The 1st mode is at 1.6Hz, the 2nd mode at 9.7Hz, and the 3rd mode at 26.7Hz. A comparison of PSD plots of the vibration data for the first and last five-second periods is shown in Figure 4, which shows that the dB-levels of higher modes in the last five-second period are significantly lower than that of the first mode. So it can be concluded that the first modal frequency is the dominant one and the first mode should be the target mode in the active vibration control.

3. CONTROL SYSTEM DESIGN

Though a sliding mode based controller often employs switching control actions, it is still possible to use this control method to suppress vibration of the flexible beam (Figure 1), as the targeted mode has a frequency which is much lower than that of the switching action employed by the controller.

To assist the control system design, we define the following control errors

$$e = y - y^d, \quad \dot{e} = \dot{y} - \dot{y}^d \quad (1)$$

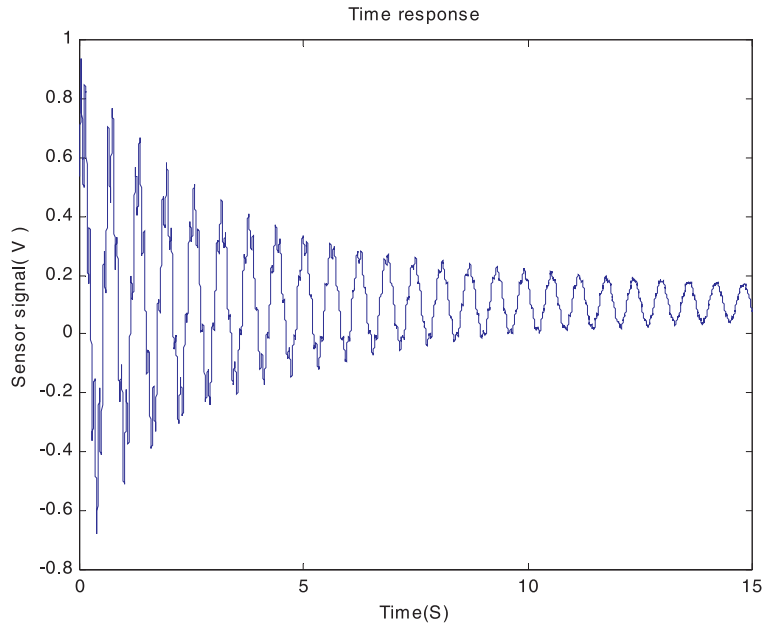


Figure 2. The time response of multi-mode excitation of the beam.

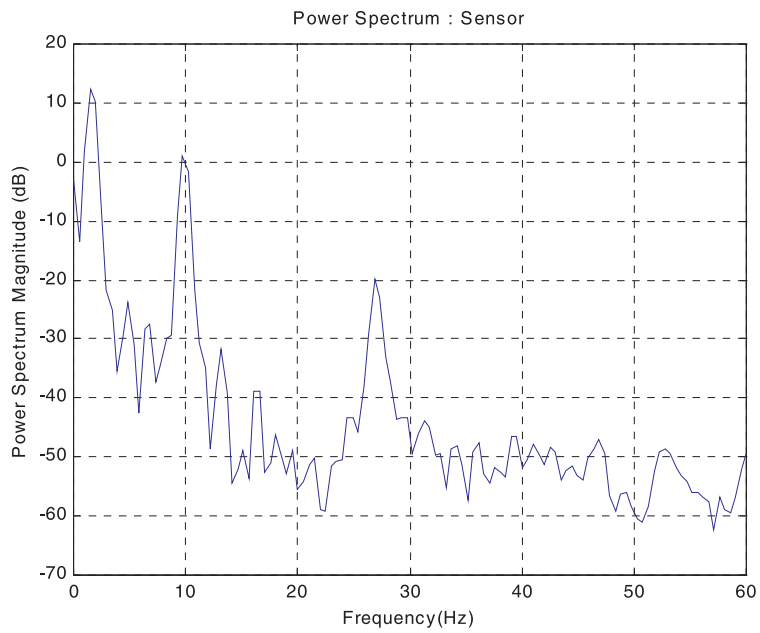


Figure 3. The power spectrum density (PSD) plot of the response of the beam.

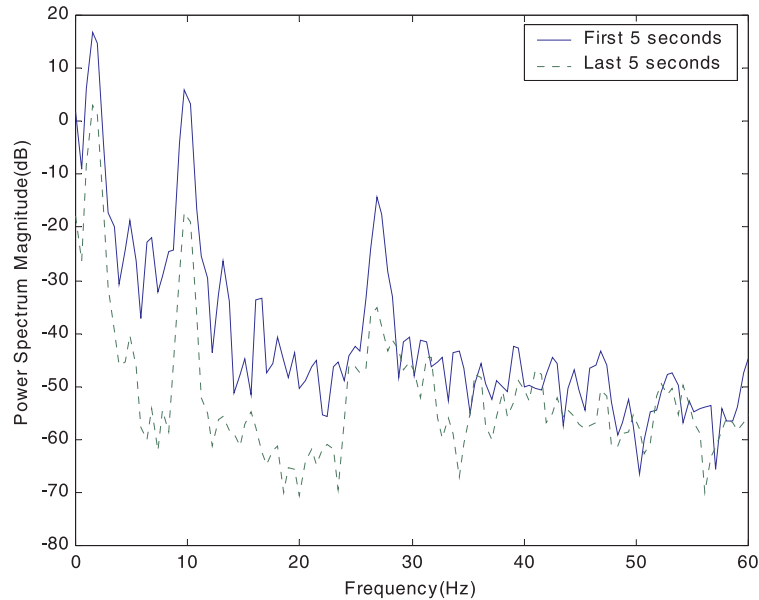


Figure 4. Comparison of the PSD plots of the first and last five-second periods.

where y and \dot{y} are the strain and strain rate; y^d and \dot{y}^d are the desired strain and strain rate, respectively. Since this is a vibration control problem, $y^d = 0$ and $\dot{y}^d = 0$. Therefore, $e = y$ and $\dot{e} = \dot{y}$. In this article, y is directly represented by the PZT sensor output in terms of voltage. The auxiliary control variables r and \dot{r} are defined by

$$r = \dot{e} + \lambda e, \quad \dot{r} = \ddot{e} + \lambda \dot{e} \quad (2)$$

where λ is a positive constant. Here, $r = 0$ functions as the sliding surface, on which the system is asymptotically stable (i.e., the control error is zero).

The proposed controller is

$$i = -K_D r - \rho \text{Tanh}(a r) \quad (3)$$

where K_D and a are positive constants. ρ is an upper bound on the uncertainties associated with flexible beam system, and is also called the Robust (R) gain in this article. A block diagram of the control system is shown in Figure 5. The functions of each control action in equation (3) will now be considered. $-K_D r$ is a linear feedback torque functioning as a Proportional plus Derivative (PD) control. Proportional control is used to decrease steady-state error and increase responsiveness of the actuator. The derivative control is to increase damping and to stabilize the actuator. $-\rho \text{Tanh}(a r)$ is a robust compensator and is used to compensate for the uncertainties of the system and to increase control accuracy and stability.

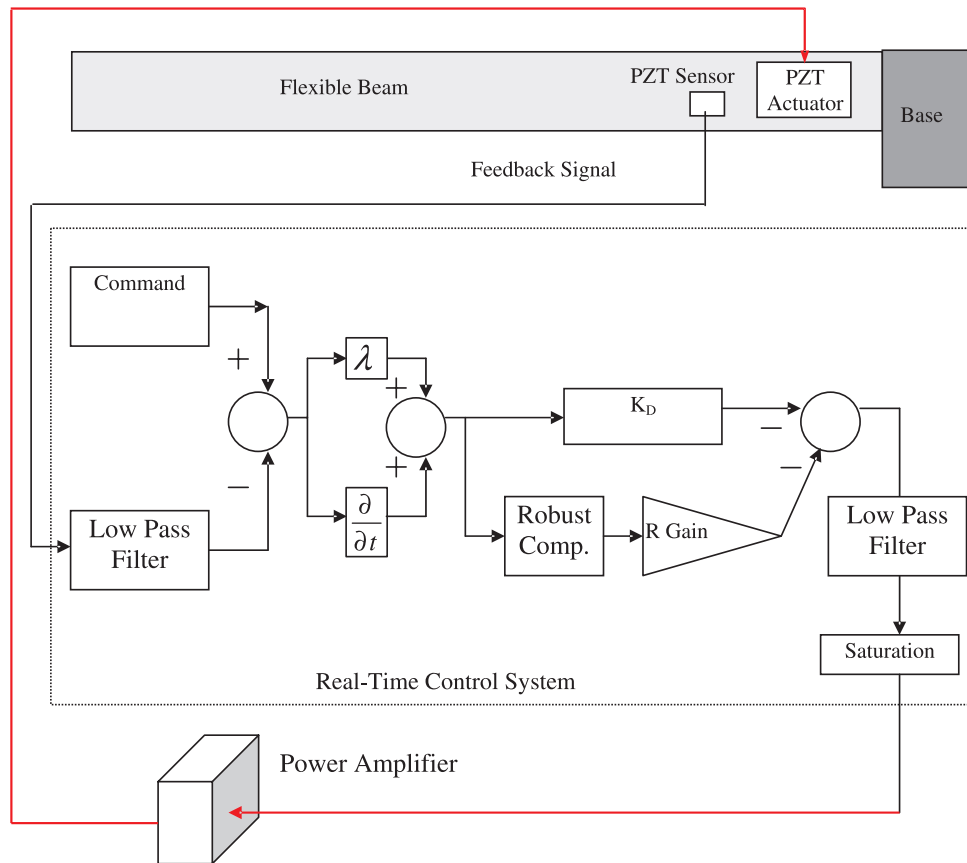


Figure 5. Block diagram of the sliding-mode control.

Remark 1

It is clear from equation (3) that the proposed controller does not need values of any plant parameters. The only information needed is the low pass filter's cut-off frequency, which is set between the first mode and the second mode to ensure that the control signal will not excite the second mode.

Remark 2

In this control approach, $r = 0$ functions as the sliding surface, on which the system is asymptotic stable, i.e., the control error is zero. In order to force the system onto the sliding surface, we employed the so-called smooth robust controller $-\rho \tanh(ar)$. The robust compensator is continuously differentiable with respect to the control variable r . It generates a smooth control action. Compared with the commonly used bang-bang or saturation robust controllers, the smooth robust controller has advantages in ensuring both smooth control input and the stability of the closed-loop system (Cai and Song, 1994). A detailed comparative study has been provided by Song and Mukherjee (1998).

Remark 3

The nonlinear nature of the *Tanh* function ensures that the compensator functions like a high gain PD controller near $r = 0$, which helps to reduce steady state error. The nonlinear nature of the *Tanh* function also ensures that its control action saturates for larger values of r , thus avoiding overshoot and oscillations. This nonlinear nature of the robust compensator distinguishes it from a Proportional plus Derivative (PD) controller.

4. EXPERIMENTAL RESULTS

To verify the effectiveness of the sliding mode based active vibration control, tests were conducted with the experimental setup described in Section 2. The first mode, at 1.6 Hz, is the target mode for vibration control. The sliding mode control (3) was implemented, along with the traditional PD control and lead compensation for comparison purposes. In each test, the beam was excited by a sinusoidal signal at its first modal frequency, combined with a white noise for the initial 5 seconds. After the initial 5 seconds, the active vibration control was implemented on the aluminum beam to suppress the vibration induced.

First, a free vibration test, where no active control is used after the initial 5-second excitation, was conducted. The signals from the PZT sensor were acquired and the time response is plotted in Figure 6. Please note that the unit of the sensor output is volts in all related figures. To verify the robustness of the proposed sliding mode controller to variation of the model parameters, vibration control of the flexible beam was conducted with additional masses added to the beam to change its modal frequencies. The same sliding mode controller is used for vibration control of the flexible with added masses for control of the beam without extra masses. To test the robustness of the sliding mode controller to the dynamics of higher modes, a multi-mode vibration control test was also conducted.

The sliding mode controller was implemented with parameters $K_d = 2$, $\rho = 7.6$, $\alpha = 2$, and $\lambda = 5$. Comparisons of the strain (which is proportional to the sensor voltage) and the strain rate (which is proportional to the derivative of sensor voltage) of the sliding mode control and free vibration are shown in Figures 7 and 8, respectively. It is clear that the beam vibration is rapidly suppressed by the sliding mode control. A comparison of the damping ratios is given in Table 3, which indicates that the sliding mode control achieves a damping ratio increase of 830% in comparison with the free vibration case. The control action for the sliding mode control is shown in Figure 9.

To further demonstrate the advantages of the sliding mode control, a traditional PD control (with $K_d = 2$, $\lambda = 5$, and $\rho = 0$) and a lead compensator $100 \frac{s+0.05}{s+4.99}$, which is designed based on a root locus approach, were implemented on the aluminum beam for the comparison purpose. To ensure a fair comparison, the PD controller uses the same parameters as the robust controller except that $\rho = 0$. For the lead compensator, a large gain is used to ensure rapid vibration suppression. The comparative results are shown in Figures 10 and 11, for the PD controller and lead compensator respectively. It is clear from these two figures that the sliding mode control is much more effective in vibration control than both the PD control and the lead compensation.

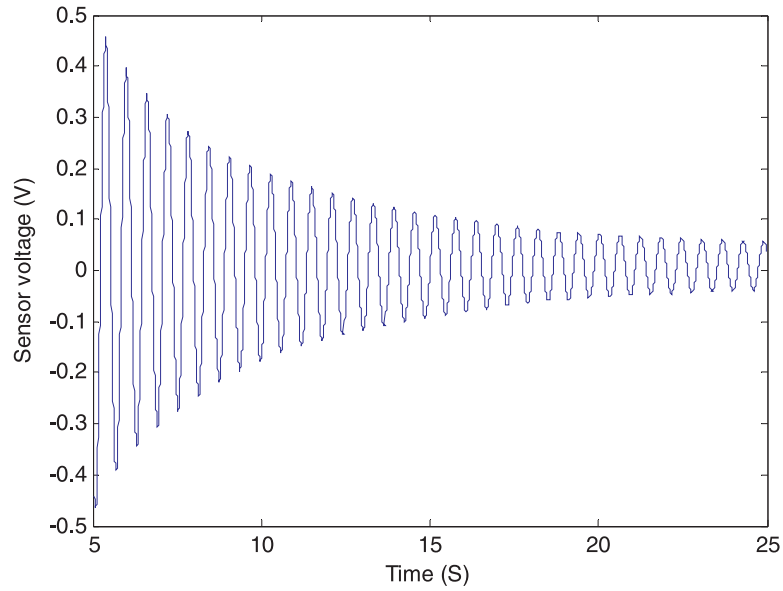


Figure 6. Free vibration of the beam.

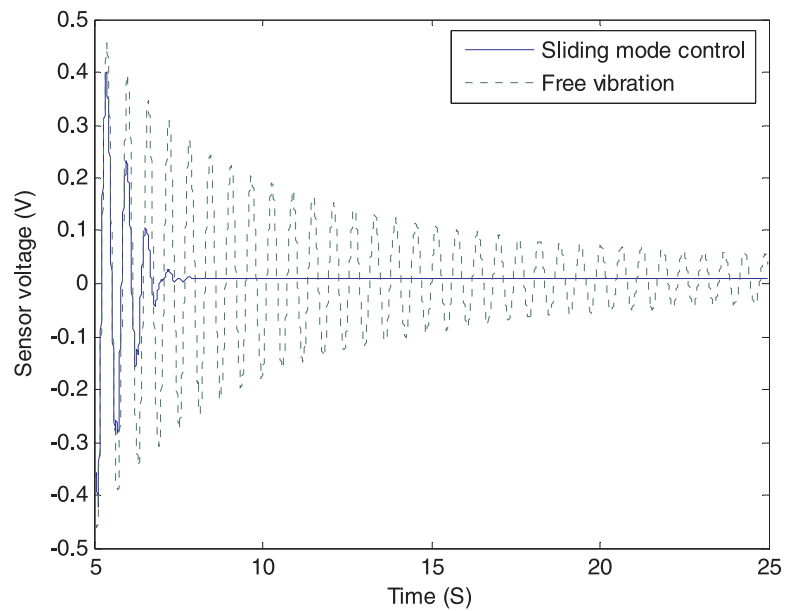


Figure 7. Time response comparison of the free vibration with the sliding mode control results.

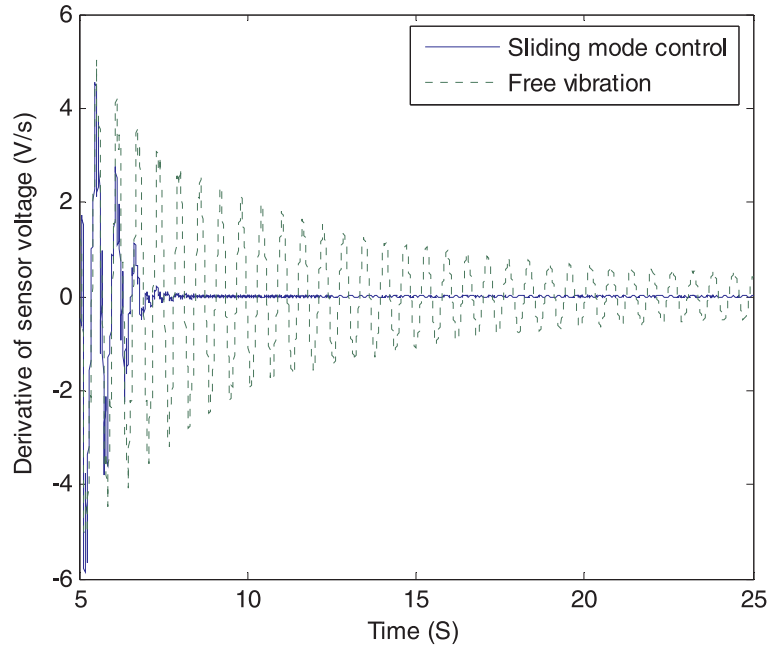


Figure 8. Comparison of strain rate errors for free vibration and vibration with sliding mode control.

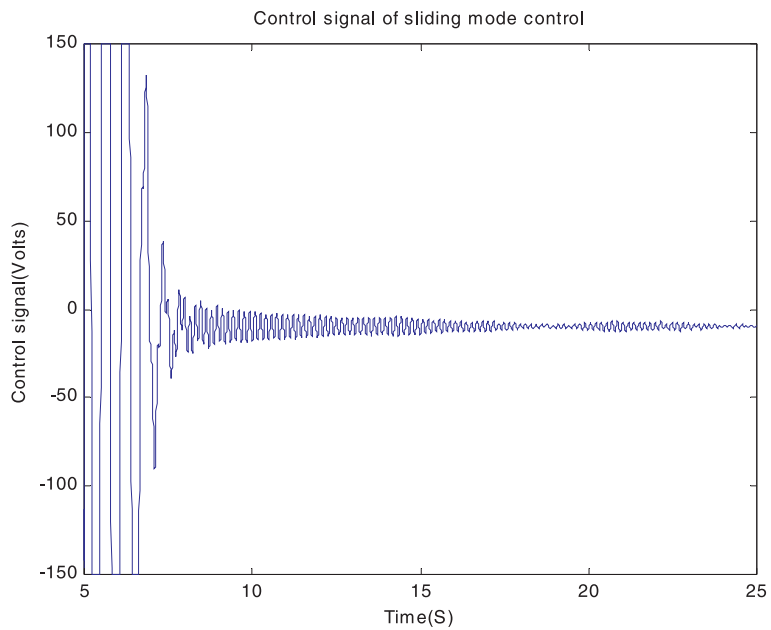


Figure 9. The control signal of the sliding mode control.

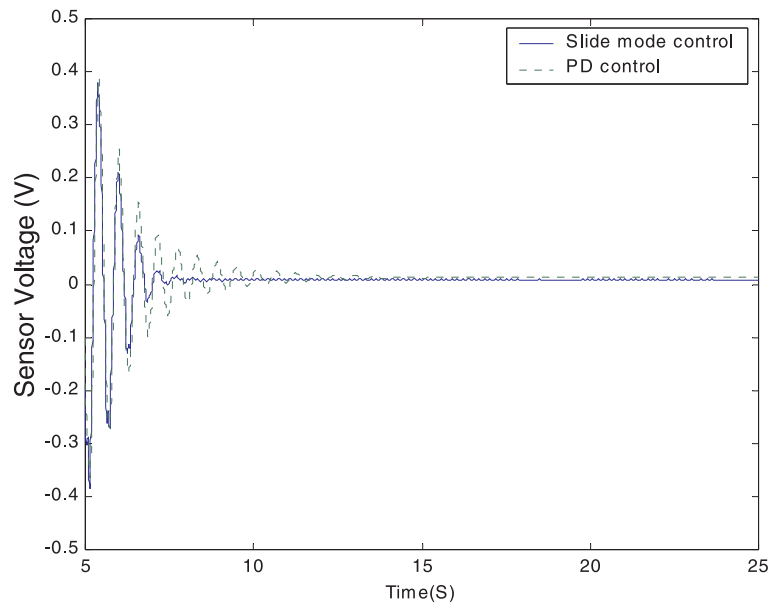


Figure 10. Time response comparison of the sliding mode control and the PD control.

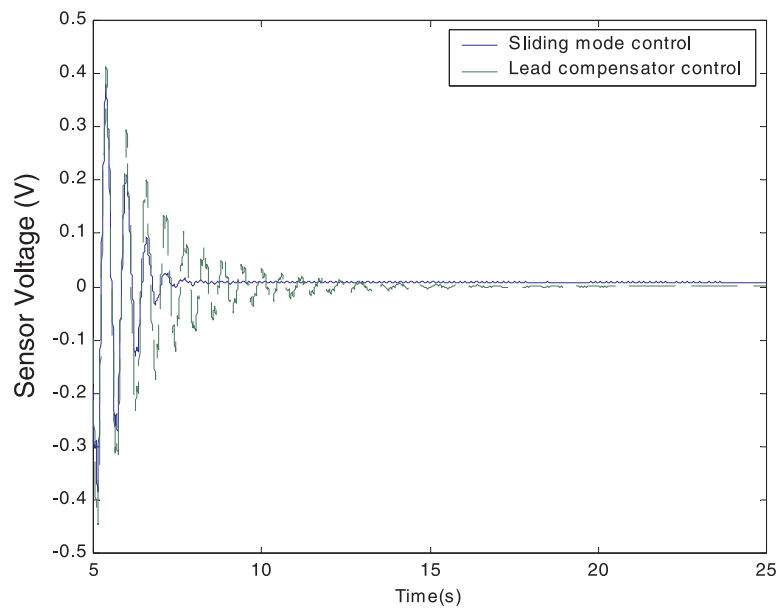


Figure 11. Time response comparison of the sliding mode control and the lead compensator.

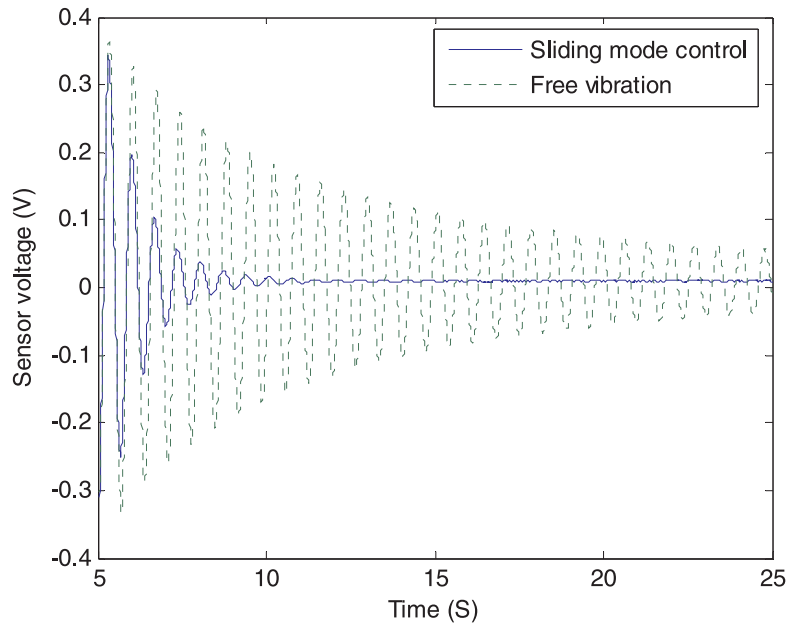


Figure 12. The time response comparison of the free vibration with the sliding mode control of the beam with added masses.

Table 3. Comparison of damping ratio for different active vibration controls.

	PD control	Lead compensation	Sliding mode control
Damping ratio achieved	0.0491	0.0240	0.1126
Damping ratio increase	0.0370	0.0119	0.1005
Damping increase (%)	305%	98%	830%

To verify the robustness of the proposed sliding mode controller, the natural frequencies of the flexible beam was changed by adding masses. Four 6-gram adhesive masses were attached to the sides of the beam, increasing the mass of the aluminum beam from 105.20 grams to 129.20 grams. This caused the first modal frequency of the beam to decrease from 1.665 Hz to 1.457 Hz. The frequency of the sinusoidal excitation signal applied was also changed to 1.457 Hz to ensure excitation of the first mode of the beam, but the same controller was used for vibration control as for the beam without added masses. From the time response comparison (Figure 12), it is clear that the sliding mode controller designed for the original beam still effectively suppresses vibrations of the beam even with the modal frequency depressed by the added masses. The experimental results demonstrate that the proposed sliding mode control is robust with respect to parametrical uncertainty of the plant.

To test its robustness with respect to disturbances of the higher order dynamics, an experiment was conducted using a multimodal excitation created by manual impact on the flexible beam (with added masses). Figure 13 shows the time response of this experiment compared with that for the free vibration, which is also caused by multi-modal excitation.

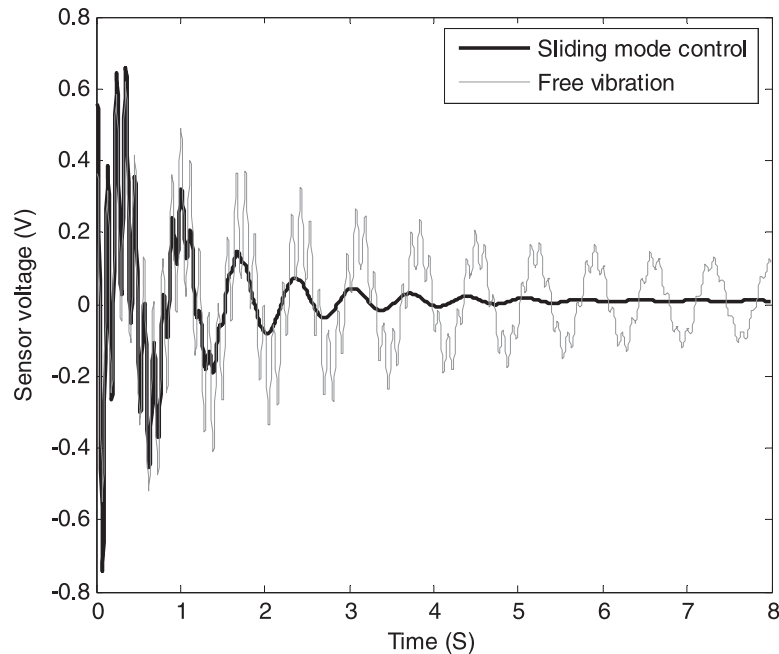


Figure 13. The time response comparison of the free vibration with the sliding-mode control when subjected to a multi-mode excitation.

Experimental results clearly show that the proposed method is still effective for vibration control with a multi-mode excitation of the flexible beam in the presence of the changed natural frequencies, further demonstrating the robustness of the proposed controller. If other types of strain actuators are used, the proposed robust controller will retain its advantages over the PD controller and the lead compensator.

5. CONCLUSIONS

In this article, a sliding mode based robust controller for the active vibration of an aluminum beam using piezoceramic sensor and actuators is designed. A PD control and a lead compensator are designed for comparison purpose. From the comparisons of the time response of the PD control and lead compensator control with the robust control, we can conclude that the robust control suppressed the vibration more effectively, in addition to requiring almost no knowledge of the flexible beam system. Further experiments on the beam with added masses and with multimodal excitation demonstrate the robustness of the proposed controller to varying model parameters and even higher modes dynamics.

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