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Active vibration suppression of a flexible beam with piezoceramic patches using robust model reference control

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Abstract
This paper presents a novel approach to active vibration suppression of a flexible beam with piezoceramic sensor and actuator. This approach employs a robust model reference controller which has the ability to handle model uncertainties. The proposed controller ensures effective active vibration suppression of the flexible beam by dramatically increasing its damping. Asymptotic stability of the closed-loop system is guaranteed as proved by Lyapunov's direct method. To demonstrate the controller's effectiveness in active vibration control and its robustness to model uncertainties, experiments, including varying the mass of the flexible beams, are conducted. Experiments show that the robust model reference control achieves rapid vibration suppression even in the presence of uncertain model parameters.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Vibration is an important aspect of many engineering systems, from machine tools to structure-borne noise in aircraft. In most cases, such vibration is undesirable and requires attenuation or vibration control. There are different active vibration control methods, such as positive position feedback control (Goh and Caughey 1985), strain rate feedback control (Pietrzakowski 2001) and optimal vibration control (Balamurugan and Narayanan 2001, Trindade *et al* 2001). Model inaccuracies or uncertainty can have adverse effects on active vibration control systems. Model reference control is a commonly used feedback control method for systems with parametric uncertainty. The general objective of model reference control is to minimize the tracking error between the output of the plant and the desired output from the reference model. The plant is assumed to have a known structure, although the parameters are unknown. The reference model should meet two requirements: (1) the reference model should reflect the performance specifications in the control tasks; (2) the ideal behavior of the reference model should be achievable for the control system. From the view of controller design, model reference control can be mainly classified as adaptive model reference control, robust model reference control and intelligent reference control.

(a) *Adaptive model reference control.* The adaptive model reference control can be classified as two distinct categories: (1) indirect model reference adaptive control and (2) direct model reference adaptive control. In direct adaptive control, the parameters of the controller are adjusted to reduce some norm of the output error between the plant and the reference model. In indirect adaptive control, the parameters of the plant are estimated and the controller is designed based on the estimated parameters, assuming that these estimated parameters are the true values of the plant parameters. Ren and Kumar (1994) studied stochastic adaptive prediction with model reference control and proposed a generalized certainty equivalence adaptive model reference control laws with simultaneous disturbance rejection. Okuno and Kase (2000) proposed the design of a model reference active control for a plant with both measurement noise and deterministic disturbances. Datta and Ho (1996) improved the performance of a standard model reference adaptive control scheme with the addition of an auxiliary tracking error feedback term.

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Limanond and Tsakalis (2000) studied the adaptive model reference control for a class of multivariable linear time-varying plants based on the polynomial equation approach.

(b) *Robust model reference control.* To enhance the robustness of the model reference control, some researchers have presented the model reference robust controller to compensate the uncertainties of the parameters of the plant and reject the disturbance. Duan *et al* (2001) proposed a robust model reference controller for multivariable linear systems with structural parameter uncertainties. The controller is decomposed into two parts: a robust state feedback stabilization controller and a robust compensation controller. The robust compensation controller compensates the effect of the state and inputs of the reference model, simultaneously minimizing the effect of uncertainties on the model reference tracking. Chen and Fukuda (1999) proposed a design technique of the robust model reference control for non-minimum phase (with respect to the relation between the disturbance and output) continuous-time dynamical systems with disturbances. Sliding-mode control is an effective approach to dealing with uncertainties and disturbances. Cunha *et al* (2003) proposed an output-feedback model-reference sliding-mode controller for linear multivariable systems based on the adaptive control formulation and on the unit vector control approach. It is shown that the closed-loop system is globally exponentially stable and the performance is insensitive to bounded input disturbances and parameter uncertainties.

(c) *Intelligent model reference control.* Intelligent control has different definitions. Generally speaking, the intelligent control is a controller that can handle various inputs, disturbances and parameter changes by using methodology from a human perspective. Artificial neural network control and fuzzy control are two typical kinds of intelligent control. Fuzzy systems have the advantage of dealing with uncertainties. Recently, fuzzy systems have been utilized in model reference controllers to enhance their performance in dealing with disturbance and uncertainties. Cho *et al* (2002) proposed an indirect model reference adaptive fuzzy control (MRAFC) which provides asymptotic tracking of a reference signal for the system with uncertain or slowly time-varying parameters. The plant is represented by the fuzzy Takagi–Sugeno model. Chen and Teng (1995) proposed a design method for an indirect model reference control structure using a fuzzy neural network. Two neural networks are utilized in the control structure. One is a fuzzy neural network controller; the other is a neural network identifier.

Due to the advantages of light weight, low cost and solid-state actuation, piezoelectric materials have been successfully applied to the vibration control as actuators and sensors. Fanson and Caughey (1990) employed positive position feedback (PPF) utilizing piezoelectric materials as actuators and sensors in the controlling of a cantilevered beam. Song *et al* (2002) successfully implemented the PPF method in the vibration control of a smart, pultruded, fiber-reinforced polymer I-beam with PZT actuator and PZT sensor. However, relatively little research has been conducted in utilizing model reference control for vibration or acoustic control with piezoelectric actuators and sensors. Clark *et al* (1993) utilized model reference control for minimizing sound radiation from vibrating structures by using piezoelectric actuators. Experimental results show that the model reference control provides a unique method for achieving a desired acoustic response without implementing microphones as error sensors. Mayhan and Washington (1998) proposed a fuzzy model reference learning control (FMRLC) to dampen the fundamental vibration mode of a cantilever beam system with a PZT actuator pair and a PZT sensor.

Table 1. Beam properties.

Symbol	Quantity	Unit	Value
L	Beam length	mm	736.5
w_b	Beam width	mm	53.1
t_b	Beam thickness	mm	1
ρ_b	Beam density	kg m^{-3}	2690
E	Modulus of elasticity	N m^{-2}	7.03×10^{10}

In this paper, a robust model reference controller is proposed to achieve active vibration control of a cantilevered flexible aluminum beam with PZT actuators and sensor. A reference model with high damping ratio and fast response is used. The model reference controller is designed to minimize the error between the designed model reference output and the real plant output. To deal with parametric uncertainties, the model reference controller includes a robust compensator. The closed-loop control system is proved to be asymptotically stable in the sense of Lyapunov. Experimental results have shown the robustness and effectiveness of the proposed method even in the presence of varying modal frequency due to changing the mass of the flexible beam.

2. Experimental set-up

An aluminum beam in a cantilevered configuration shown in figure 1 is used as the experimental object to test the effectiveness of the proposed vibration suppression method. PZT type piezoelectric ceramic patches are used as the smart sensor and actuator. The physical position of the piezoelectric actuators and sensor is shown in figure 2. Two PZT patches are surface-bonded on the lateral surfaces of the aluminum beam. These two patches are used as actuators to excite and to enable active vibration control of the beam. One PZT patch is bonded on the lateral surface of the beam and acts as a sensor for the feedback signals in the active control algorithms. The properties of the PZT patches are shown in table 2. The vibration suppression algorithm is designed in Matlab/Simulink and then downloaded to the dSPACE digital data acquisition and real-time control system to implement the proposed control algorithm. The dSPACE system has an analog-to-digital converter and a digital-to-analog converter. In the hosting computer, the dSPACE ControlDesk module is used to develop a graphical user interface (GUI) for real-time control and data acquisition.

3. Design of model reference controller and stability proof

3.1. Design of the controller

A model reference controller shown in figure 3 will be designed to control the vibration of a flexible beam (the plant)

Table 2. Properties of PZT patches used on the beam.

Symbol	Quantity	Unit	PZT actuator	PZT sensor
$l \times w \times t$	Dimensions	mm	$46 \times 33.27 \times 0.25$	$14 \times 7 \times 0.25$
d_{33}	Strain coefficient	C N^{-1}	7.41×10^{-10}	7.41×10^{-10}
d_{31}	Strain coefficient	C N^{-1}	-2.74×10^{-10}	-2.74×10^{-10}
ρ_P	PZT density	kg m^{-3}	7500	7500
E_P	Young's modulus	N m^{-2}	6.3×10^{10}	6.3×10^{10}

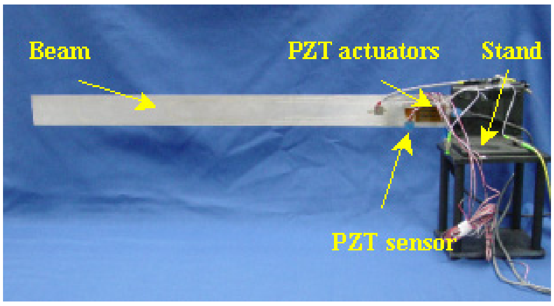

Figure 1. The experimental set-up.

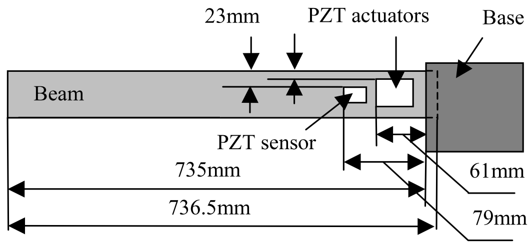

Figure 2. The physical position of the piezoelectric actuators and sensor.

with a PZT sensor and PZT actuators. To achieve vibration control, the reference model will have a much higher damping ratio than that of the plant. To ensure a quicker response, the natural frequency of the reference model will be higher than that of the plant. It is also assumed that the exact values of the plant parameters are not known for control design. The model reference controller will be designed based on the estimated values.

Experiments show that the dominant mode of the flexible beam is its first mode and is the major concern for vibration suppression (details can be found in section 4). The damping ratio for the beam is $\zeta = 0.011\,94$ and the natural frequency of the plant is $\omega_n = 10.05$ rad s^{-1}. The transfer function of the flexible beam at this dominant mode can be represented by

$$\frac{X_1(S)}{U(S)} = \frac{12.5}{S^2 + 0.24S + 101}, \tag{1}$$

where $X_1(S)$ is the Laplace transform of the PZT sensor output $x_1(t)$, which is proportional to the local strain, and $U(S)$ is the Laplace transform of the actuation signal $u(t)$ for the PZT actuator.

The state-space model of the flexible beam can be found as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u, \tag{2}$$

where

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \qquad \boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -101 & -0.24 \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} 0 \\ b \end{bmatrix} \tag{3}$$

where $b = 12.5$, which is the gain of the plant (1), and $x_2 = \dot{x}_1$, which is proportional to the strain rate.

Please note that the exact values in (3) are not available for the control design. Instead, only the estimates of these values will be available for the control design.

To achieve a better damping and quick response of the closed-loop system, the damping ratio and the natural frequency of the reference model will be chosen larger than those of the original plant. The damping ratio of the reference model is chosen as 0.3934, which is significant higher than that of the original plant, and the natural frequency of the reference model is chosen as 23.79 rad s^{-1}. Based on these two values, the reference model is chosen as

$$\frac{X_{1d}(S)}{V(S)} = \frac{8.5}{S^2 + 18.72S + 566} \tag{4}$$

where $X_{1d}(S)$ is the Laplace transform of $x_{1d}(t)$, the desired output of the reference model, and $V(S)$ is the Laplace transform of $v(t)$, the command input.

The state-space representation of the reference model (4) is

$$\dot{\boldsymbol{x}}_d = \boldsymbol{A}_{ref}\boldsymbol{x}_d + \boldsymbol{B}_{ref}v \tag{5}$$

where

$$\boldsymbol{x}_d = \begin{bmatrix} x_{1d} \\ x_{2d} \end{bmatrix}, \qquad \boldsymbol{A}_{ref} = \begin{bmatrix} 0 & 1 \\ -566 & -18.72 \end{bmatrix}, \qquad \boldsymbol{B}_{ref} = \begin{bmatrix} 0 \\ b_{ref} \end{bmatrix}, \tag{6}$$

where $b_{ref} = 8.5$, which is the gain of the reference model (4), and $x_{2d} = \dot{x}_{1d}$.

To assist the control design, define

$$\boldsymbol{e} = \boldsymbol{x}_d - \boldsymbol{x} = \begin{bmatrix} x_{1d} - x_1 \\ x_{2d} - x_2 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}. \tag{7}$$

Based on (2) and (5), the error dynamics can be derived as

$$\begin{aligned} \dot{\boldsymbol{e}} &= \dot{\boldsymbol{x}}_d - \dot{\boldsymbol{x}} \\ &= \boldsymbol{A}_{ref}\boldsymbol{x}_d + \boldsymbol{B}_{ref}v - \boldsymbol{A}\boldsymbol{x} - \boldsymbol{B}u \\ &= \boldsymbol{A}_{ref}(\boldsymbol{x}_d - \boldsymbol{x} + \boldsymbol{x}) - \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_{ref}v - \boldsymbol{B}u \\ &= \boldsymbol{A}_{ref}\boldsymbol{e} + (\boldsymbol{A}_{ref} - \boldsymbol{A})\boldsymbol{x} + \boldsymbol{B}_{ref}v - \boldsymbol{B}u. \end{aligned} \tag{8}$$

Define

$$\tilde{\boldsymbol{A}} = \boldsymbol{A}_{ref} - \boldsymbol{A} = \begin{bmatrix} 0 & 0 \\ m & n \end{bmatrix}. \tag{9}$$

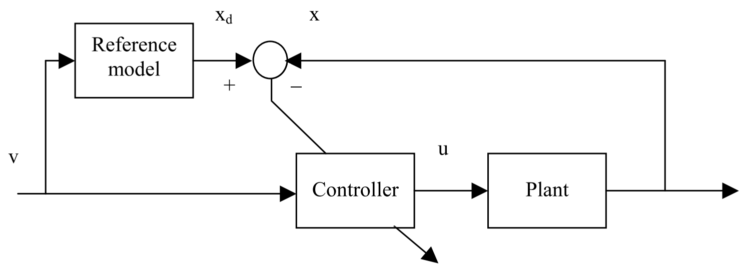

Figure 3. The block diagram of the model reference control system.

Since A is not exactly known, m and n will also not be exactly known to control design. With definition (9), (8) can be rewritten as

$$\dot{e} = A_{\text{ref}} e + \tilde{A} x + B_{\text{ref}} v - B u. \tag{10}$$

In a vibration control problem, the command input is set to zero, i.e.

$$v = 0.$$

Therefore the error dynamics (10) can be represented by

$$\dot{e} = A_{\text{ref}} e + \tilde{A} x - B u. \tag{11}$$

To assist the control design, define

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{12}$$

Solving for $P (P > 0)$ from the Lyapunov equation with Q defined in (12),

$$A_{\text{ref}}^{\text{T}} P + P A_{\text{ref}} = -Q \tag{13}$$

gives

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 15.1608 & 0.0009 \\ 0.0009 & 0.0268 \end{bmatrix} > 0. \tag{14}$$

The values of p_{12} and p_{22} will be used later for the control design.

To further assist the control design, several items associated with uncertainties will be defined. Though the gain b is not exactly known, its upper bound and lower bound are assumed known:

$$0 < b_{\min} \leqslant b \leqslant b_{\max}. \tag{15}$$

The estimate $\hat{b}$ of gain b is chosen as

$$\hat{b} = \sqrt{b_{\min} b_{\max}}. \tag{16}$$

To help to characterize the uncertainties caused by unknown parameters m and n, define

$$f = m x_1 + n x_2 - \hat{m} x_1 - \hat{n} x_2, \tag{17}$$

where f is the uncertainty caused by unknown parameters m and n, and $\hat{m}$ and $\hat{n}$ are the estimation of m and n.

The f is bounded by

$$|f| \leqslant |m - \hat{m}|\,|x_1| + |n - \hat{n}|\,|x_2| = F, \tag{18}$$

where F is the upper bound on the uncertainty f.

Define

$$\hat{u} = \hat{m} x_1 + \hat{n} x_2. \tag{19}$$

The control law u is given by

$$u = -\hat{b}^{-1} \left(\hat{u} + \rho \operatorname{sgn}(e_1 p_{12} + e_2 p_{22}) \right), \tag{20}$$

with

$$\rho = \beta(F + \eta) + (\beta - 1)\sigma, \tag{21}$$

where sgn is the sgn function ($\operatorname{sgn}(s) = 1$, if $s > 0$; $\operatorname{sgn}(s) = -1$, if $s < 0$); α and η are positive numbers; σ is the upper bound of $|\hat{u}|$; and β is defined as

$$\beta = \sqrt{\frac{b_{\max}}{b_{\min}}}. \tag{22}$$

Remark 1. The term $\hat{u}$ (19) represents the control action designed based on the estimated value of m and n. Physically, it is a proportional plus derivative (PD) controller, which alone will not be able to ensure the stability of the closed-loop system in the presence of uncertainties.

Remark 2. ρ defined in (21) is the upper bounding function for the overall uncertainties, which are caused by inexactly known parameters b, m and n.

Remark 3. The term $\rho \operatorname{sgn}(e_1 p_{12} + e_2 p_{22})$ in (20) is a variable structure robust compensator to deal with the overall uncertainties and ensure stability of the closed-loop system.

3.2. Stability proof of the controller

In this section, the Lyapunov direct method will be used to prove the asymptotical stability of the closed-loop system. Define the Lyapunov function candidate as

$$V(t) = e^{\text{T}} P e, \tag{23}$$

where P is given in equation (14). It is clear that $V(t)$ is positive definite.

From equations (23) and (11), we can derive the time derivative of V along the closed-loop system trajectory as

$$\begin{aligned} \dot{V}(t) &= \dot{e}^{\text{T}} P e + e^{\text{T}} P \dot{e} \\ &= (e^{\text{T}} A_{\text{ref}}^{\text{T}} + x^{\text{T}} \tilde{A}^{\text{T}} + v B_{\text{ref}}^{\text{T}} - u B^{\text{T}}) P e \\ &\quad + e^{\text{T}} P (A_{\text{ref}} e + \tilde{A} x + B_{\text{ref}} v - B u) \\ &= e^{\text{T}} (A_{\text{ref}}^{\text{T}} P + P A_{\text{ref}}) e + 2 e^{\text{T}} P (\tilde{A} x + B_{\text{ref}} v - B u) \end{aligned}$$

which can be represented by

$$\dot{V}(t) = \boldsymbol{e}^{\mathrm{T}}(\boldsymbol{A}_{\mathrm{ref}}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_{\mathrm{ref}})\boldsymbol{e} + 2M \tag{24}$$

where

$$M = \boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}(\tilde{\boldsymbol{A}}\boldsymbol{x} + \boldsymbol{B}_{\mathrm{ref}}v - \boldsymbol{B}u). \tag{25}$$

Substituting (13) into (24) gives

$$\dot{V}(t) = -\boldsymbol{e}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{e} + 2M. \tag{26}$$

Furthermore, M can be expressed as

$$M = [\,e_1 \quad e_2\,]\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \times \left(\begin{bmatrix} 0 & 0 \\ m & n \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_{\mathrm{ref}} \end{bmatrix} v - \begin{bmatrix} 0 \\ b \end{bmatrix} u\right) \tag{27}$$

$$M = (e_1 p_{12} + e_2 p_{22})(mx_1 + nx_2 + b_{\mathrm{ref}}v - bu). \tag{28}$$

In a vibration control problem, the command input is set to zero, i.e. $v = 0$. Therefore, M can be expressed as

$$M = (e_1 p_{12} + e_2 p_{22})(mx_1 + nx_2 - bu). \tag{29}$$

Substituting the uncertainty (17) and control law (20) into (29) results in

$$\begin{aligned} M &= (e_1 p_{12} + e_2 p_{22})(\hat{m}x_1 + \hat{n}x_2 + f \\ &\quad - b\hat{b}^{-1}(\hat{m}x_1 + \hat{n}x_2 + \rho\,\mathrm{sgn}(e_1 p_{12} + e_2 p_{22}))) \\ M &= (e_1 p_{12} + e_2 p_{22})((1 - b\hat{b}^{-1})(\hat{m}x_1 + \hat{n}x_2) + f \\ &\quad - \rho b\hat{b}^{-1}\,\mathrm{sgn}(e_1 p_{12} + e_2 p_{22})) \end{aligned} \tag{30}$$

where the upper bounding function ρ is defined in (21), based on which the following relationship can be derived:

$$\begin{aligned} \rho &\geqslant \beta(F + \eta) + (\beta - 1)\left|\hat{u}\right| \\ &\geqslant \beta(f + \eta) + (\beta - 1)\left|\hat{m}x_1 + \hat{n}x_2\right| \\ &\geqslant \eta\hat{b}b^{-1} + \left|(\hat{b}b^{-1} - 1)(\hat{m}x_1 + \hat{n}x_2) + \hat{b}b^{-1}f\right|. \end{aligned} \tag{31}$$

Equations (30) and (31) clearly explain the motivation to define ρ in the form of (21), which upper bounds the uncertainties in (30) as shown in (31).

From (30) and (31), the following relationship can be derived:

$$\begin{aligned} M &\leqslant (e_1 p_{12} + e_2 p_{22})((1 - b\hat{b}^{-1})(\hat{m}x_1 + \hat{n}x_2) + f \\ &\quad - (\eta + |(1 - b\hat{b}^{-1})(\hat{m}x_1 + \hat{n}x_2) + f|) \\ &\quad \times \mathrm{sgn}(e_1 p_{12} + e_2 p_{22})). \end{aligned} \tag{32}$$

Define

$$s = e_1 p_{12} + e_2 p_{22}$$

and

$$a = (1 - b\hat{b}^{-1})(\hat{m}x_1 + \hat{n}x_2) + f.$$

Now (32) can be expressed as

$$\begin{aligned} M &\leqslant s\,(a - (\eta + |a|)\,\mathrm{sgn}(s)) \\ M &\leqslant sa - (\eta + |a|)\,|s| < 0, \qquad \text{if } \boldsymbol{e} \neq \boldsymbol{0}. \end{aligned} \tag{33}$$

Therefore, combining (26) and (33) gives

$$\dot{V}(t) = -\boldsymbol{e}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{e} + 2M < 0, \qquad \text{if } \boldsymbol{e} \neq \boldsymbol{0}. \tag{34}$$

Therefore, the closed-loop system is asymptotically stable based on Lyapunov's direct method.

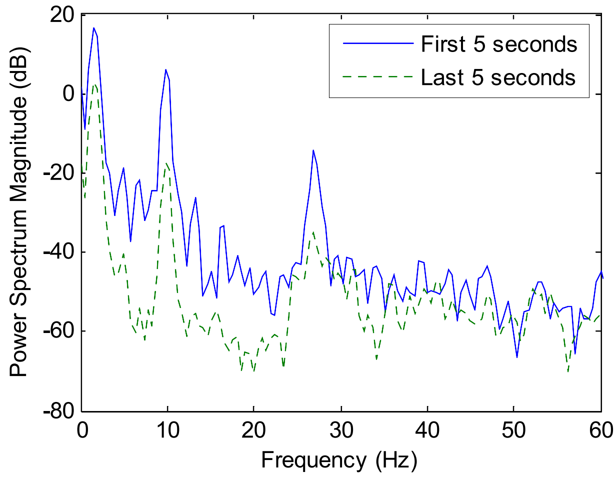

Figure 4. Comparison of PSD plots of first 5 s and last 5 s.

Remark 4. The sgn function is usually used in a robust control to guarantee the asymptotically stability of the closed-loop system. However, the sgn function term often leads to chattering. Therefore it is more advantageous to use the smooth hyperbolic tangent function to reduce the chattering often associated with robust controllers (Song and Mukherjee 1998). The trade-off is that the hyperbolic tangent function cannot guarantee asymptotic stability. The system trajectory in this case will be uniformly ultimately bounded (Song and Mukherjee 1998).

4. Experimental results

Before the implementation of the control method, open-loop testing of the aluminum beam is carried out to find the dominant modal frequency that will be the target for the vibration control.

4.1. Open-loop testing of the aluminum beam

First, an open-loop testing is performed to find the dominant mode of the beam for active vibration control. The beam is subjected to an impact for a multi-mode excitation and the data was recorded for 15 s. A power spectrum density (PSD) plot of the time response of the multi-mode excitation reveals the modal frequencies of the system: first mode at 1.64 Hz, second mode at 9.77 Hz and the third mode at 26.85 Hz. A comparison of PSD plots of the vibration data for the first 5 s and the last 5 s is shown in figure 4. The dB value and dB drop of the first three modes are shown in table 3. From figure 4 and table 3, it can be seen that the dB levels of the second and third modes at the first 5 s and last 5 s periods are significantly lower than that of the first mode. The dB drop in value of the first three modes shows that the second and third modes attenuated much faster than the first mode. Therefore, it can be concluded that the first modal frequency, 1.64 Hz, is the dominant one and that the first mode should be the target mode in the active vibration control.

Table 3. PSD drop comparison of first 5 s and last 5 s.

	Mode 1	Mode 2	Mode 3
dB of first 5 s	16.73	5.98	−14.36
dB of last 5 s	2.91	−17.23	−35.19
dB drop	13.82	23.21	20.83

4.2. Experimental result for model reference control

Three experiments were conducted: free vibration, model reference control and model reference control of the beam with increased mass. The first mode is the target mode for vibration control. In each test, the beam is excited by a sinusoidal signal at its first modal frequency combined with white noise for the initial 5 s. After the initial 5 s, the active vibration control is introduced to suppress the induced vibration. Free vibration is realized by applying no control action to the beam after the initial 5 s excitation.

In the real-time implementation of the proposed model reference control, the parameters of the controller are chosen by the method described in section 3. $\hat{m}$ ($\hat{m} = 465$) and $\hat{n}$ ($\hat{n} = 18.48$) are the estimates of the m and n in (9); $|m - \hat{m}| \leqslant \mu$, $|n - \hat{n}| \leqslant \nu$, where μ ($\mu = 10$) and ν ($\nu = 2$) are the upper bound of the estimation error of $\hat{m}$ and $\hat{n}$:

$$|x_1| \leqslant |x_1|_{\max}, \qquad \text{where } |x_1|_{\max} = 0.5 \tag{35}$$

and

$$|x_2| \leqslant |x_2|_{\max}, \qquad \text{where } |x_2|_{\max} = 5. \tag{36}$$

To calculate the upper bound of uncertainty in (18),

$$|f| \leqslant |m - \hat{m}||x_1| + |n - \hat{n}||x_2| \leqslant \mu|x_1|_{\max} + \nu|x_2|_{\max} = F \tag{37}$$

$$F = 10 \times 0.5 + 2 \times 5 = 15. \tag{38}$$

The upper bound and lower bound of b are given as

$$b_{\max} = 12.9 \qquad \text{and} \qquad b_{\min} = 11$$
$$\beta = \sqrt{\frac{b_{\max}}{b_{\min}}} = \sqrt{\frac{12.9}{11}} = 1.0829. \tag{39}$$

Choose $\eta = 0.1$.

To satisfy (21)

$$\rho = \beta(F + \eta) + (\beta - 1)\sigma$$
$$\rho = \beta(F + \eta) + (\beta - 1)(\hat{m}|x_1|_{\max} + \hat{n}|x_2|_{\max})$$
$$\rho = 1.0829(15 + 0.1) + 0.0829 \times (465 \times 0.5 + 18.48 \times 5)$$
$$\rho = 43.2860. \tag{40}$$

Therefore, $\rho \geqslant 43.2860$ will guarantee the stability of the control system.

With the above calculated parameters, the control law (20) becomes

$$u = -\frac{1}{12.5}\{(-465)x_1 - 18.48x_2 + 43.2860\,\mathrm{sgn}(0.0009e_1 + 0.0268e_2)\}. \tag{41}$$

To reduce the chattering which may happen in the conduct of this method, a hyperbolic tangent function is applied in the experiment to reduce the possible chattering. The parameter ρ is increased to 100 in the experiment to get a satisfactory result:

$$u = -\frac{1}{12.5}\{(-465)x_1 - 18.48x_2 + 100\tanh[10(0.0009e_1 + 0.0268e_2)]\}$$

which is

$$u = 37.2x_1 + 1.4784x_2 - 8\tanh(0.009e_1 + 0.268e_2). \tag{42}$$

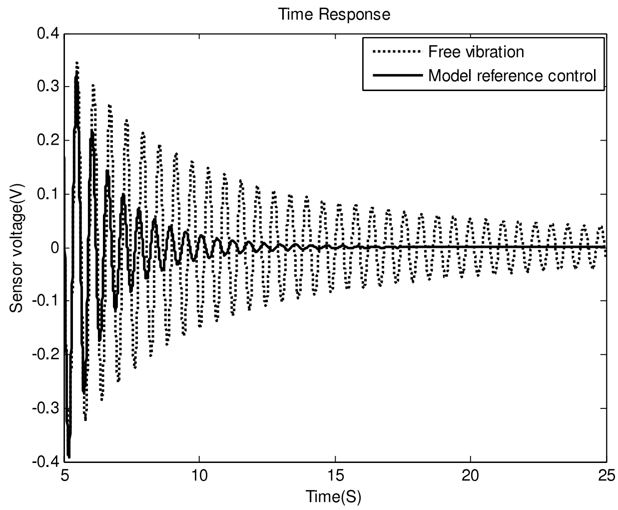

Figure 5. Comparison of time response of free vibration with model reference control.

From the time response comparison of the free vibration with the model reference control of the beam (figure 5), the vibration is totally suppressed in about 17 cycles (10 s) with the proposed robust model reference control. This means the dominant mode of the vibration has successfully been suppressed. No higher modes have been excited.

To verify the robustness of the proposed model reference control approach, the plant is changed by adding mass. Four 6 g adhesive masses are attached to both sides of the beam. The mass of the aluminum beam is increased from 105.20 to 129.20 g. By adding the mass on the flexible beam, the first modal frequency of the original beam is decreased from 1.665 to 1.457 Hz. The increase in mass has caused the decrease of the natural frequency of the beam. Therefore, in this experiment, the frequency of the sinusoidal excitation signal has been changed to 1.457 Hz to ensure excitation of the beam.

The same controller (42) is used for the vibration control of the beam with added mass. From the time response comparison (figure 6), it is clear that the same robust model reference controller designed for the original beam still effectively suppresses vibrations of the beam with a lower modal frequency caused by adding mass. The experimental results demonstrate that the proposed model reference control is robust to the parameter uncertainty of the plant.

To test its robustness to higher-order dynamics and disturbances, an experiment of the proposed controller with a multi-modal excitation by manual impacting the flexible beam with added mass is conducted. Figure 7 shows the comparison of time response of this experiment with that of free vibration.

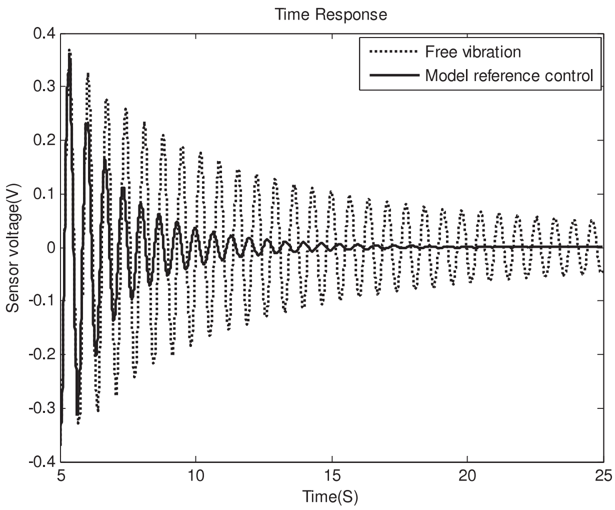

Figure 6. Comparison of time response of free vibration with model reference control with added mass.

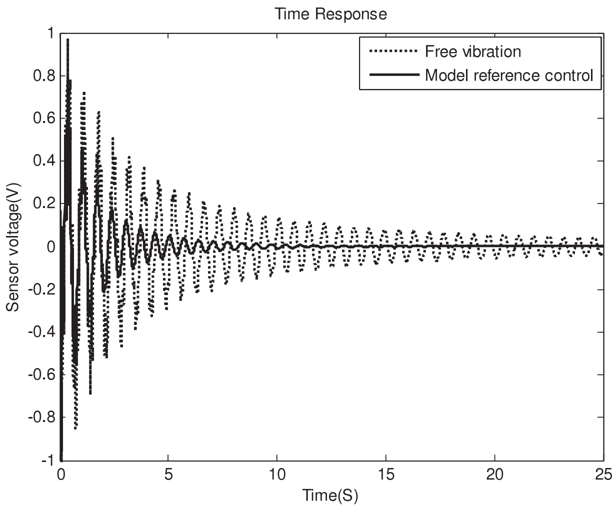

Figure 7. Time response comparison of free vibration with model reference control for multi-mode excitation.

Experimental results clearly show that the proposed method is still effective in vibration control and this further demonstrates the robustness of the proposed controller.

5. Conclusion

In this paper, a robust model reference controller is developed for active vibration suppression of a flexible beam with piezoceramic sensor and actuator. For the purpose of vibration control, the reference model is designed so that it has a much higher damping ratio and faster response, as compared with those of the original plants. To deal with parametric uncertainties, a robust compensator is used in the proposed model reference control. Asymptotical stability of the closed-loop system is proved using Lyapunov's direct method. Experiments conducted on the flexible beam clearly show the effectiveness of the proposed controller for active vibration suppression. Further experiments on the beam with added mass and with multi-modal excitation demonstrate the robustness of the proposed controller to varying model parameters and even higher modes dynamics.

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References

Balamurugan V and Narayanan S 2001 Active vibration control of smart shells using distributed piezoelectric sensors and actuators *Smart Mater. Struct.* **10** 173–80

Chen X and Fukuda T 1999 Model reference robust control for non-minimum phase system with disturbances *Int. J. Control* **72** 123–32

Chen Y and Teng C 1995 A model reference control structure using a fuzzy neural network *Fuzzy Sets Syst.* **73** 291–312

Cho Y, Park C and Park M 2002 An indirect model reference adaptive fuzzy control for SISO Takagi–Sugeno model *Fuzzy Sets Syst.* **131** 197–215

Clark R L, Gibbs G P and Fuller C R 1993 An experimental study implementing model reference active structural acoustic control *J. Acoust. Soc. Am.* **93** 3258–64

Cunha J P V S, Hsu L, Costa R R and Lizarralde F 2003 Output-feedback model-reference sliding mode control of uncertain multivariable systems *IEEE Trans. Autom. Control* **48** 2245–50

Datta A and Ho M 1996 A modified model reference adaptive control scheme for rigid robots *IEEE Trans. Robot. Autom.* **12** 466–70

Duan G R, Liu W Q and Liu G P 2001 Robust model reference control for multivariable linear systems subject to parameter uncertainties *Proc. Inst. Mech. Eng.* I **215** 599–610

Fanson L J and Caughey K T 1990 Positive position feedback control for large space structures *AIAA J.* **28** 717–24

Goh C and Caughey T K 1985 On the stability problem caused by infinite actuator dynamics in the collocated control of large space structure *Int. J. Control* **41** 787–802

Limanond S and Tsakalis K S 2000 Model reference adaptive and nonadaptive control of linear time-varying plants *IEEE Trans. Autom. Control* **45** 1290–300

Mayhan P and Washington G 1998 Fuzzy model reference learning control: a new paradigm for smart structures *Smart Mater. Struct.* **7** 874–84

Okuno H and Kase W 2000 A design of model reference adaptive control systems with unknown disturbances *Electr. Eng. Japan* **131** 1633–8

Pietrzakowski M 2001 Active damping of beams by piezoelectric systems: effects of bonding layer properties *Int. J. Solids Struct.* **38** 7885–97

Ren W and Kumar P R 1994 Stochastic adaptive prediction and model reference control *IEEE Trans. Autom. Control* **39** 2047–60

Song G and Mukherjee R 1998 A comparative study of conventional nonsmooth time-invariant and smooth time-varying robust compensators *IEEE Trans. Control Syst. Technol.* **6** 571–6

Song G, Qiao P, Sethi V and Prasad A 2002 Active vibration control of a smart pultruded fiber-reinforced polymer I-beam *Proc. SPIE Int. Symp. on Smart Structures and Materials (San Diego, CA, 2002)*

Trindade A M, Benjeddou A and Ohayon R 2001 Piezoelectric active vibration control of damped sandwich beams *J. Sound Vib.* **246** 653–77